Convex Optimization for Robust Trajectory Planning and Control

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Autonomous Controls Lab

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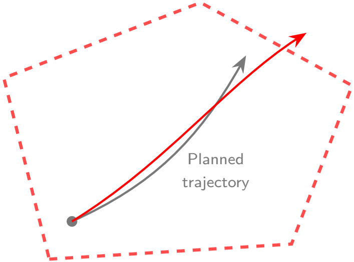
Overview

- **Numerical trajectory planning** generates **dynamically feasible trajectories** ensuring **prescribed constraints**
- In practice,
  - **exogenous** perturbations (e.g. winds)
  - **endogenous** perturbations (e.g. actuator misalignment)

  cause **deviations** from the **planned** trajectory

  $\Longrightarrow$ **constraints** may be then **violated**.



Q0: How to incorporate **robust constraint satisfaction**?

Overview

Autonomous Controls Lab

Robust trajectory optimization leads to **Semi-Infinite Programming (SIP)** problems.

In discrete-time, SIP have:

- **finite** number of decision variables;
- **finite** number of time instances at which constraints are enforced;
- **infinite** number of uncertainty realizations;

$\Rightarrow$ **infinite** number of constraints.

Overview

	Uncertainty characterization
Sampling-based [19], [17]	Compact set
Tube-based [11], [5], [3], [15]	Polytopic set
Min-max MPC [10], [18]	Class-$\mathcal{K}$ functions
Stochastic planning [9], [4]	Probabilistic
Funnel synthesis [6],[7]	L^∞-norm bound
System-Level Synthesis [1], [8]	Ellipsoidal set

Table: Some techniques for robust trajectory planning and control

Contents

Problem Formulation

Dynamics

- Discrete-time linear system with endogenous perturbations n_t.
- Dynamics **with perturbations** in recursive form:

$$x_{t+1} = A_t x_t + B_t u_t + E_t n_t \qquad t = 0, \ldots, N$$

- Variables:

$$\bar{u}_t := \begin{bmatrix} u_0 \\ \vdots \\ u_t \end{bmatrix} \quad \bar{n}_t := \begin{bmatrix} n_0 \\ \vdots \\ n_t \end{bmatrix}$$

- Dynamics in **stacked form**:

$$x_{t+1} = \Phi_{t+1,0} x_0 + \bar{B}_t \bar{u}_t + \bar{E}_t \bar{n}_t$$

Problem Formulation

Nominal and deviation dynamics

- We decompose **state** into **nominal** x^n and **deviation states** Δx:

$$x_{t+1} = x^n_{t+1} + \Delta x_{t+1}$$

- Nominal state and deviation **dynamics** in **recursive form**:

$$x^n_{t+1} = A_t x^n_t + B_t u_t,$$
$$\Delta x_{t+1} = A_t \Delta x_t + E_t n_t.$$

- Nominal state and deviation **dynamics** in **stacked form**:

$$x^n_{t+1} = \Phi_{t+1,0} x^n_0 + \bar{B}_t \bar{u}_t$$
$$\Delta x_{t+1} = \Phi_{t+1,0} \Delta x_0 + \bar{E}_t \bar{n}_t$$

Problem Formulation

Perturbation, Cost and Constraints

- **Nominal state and control**-dependent perturbation:

$$0 \leq n_t \leq f(x_t^n, u_t)$$

  where $f(x_t^n, u_t)$ is **convex and elementwise nonnegative**.

- **Convex** cost function

$$J(\bar{u}_T)$$

- **SIP time-varying** state constraints

$$H_{t+1}x_{t+1} \leq h_{t+1} \qquad \forall \bar{n}_t, \quad t = 0, \ldots, N$$

Problem Formulation

Trajectory planning problem

Semi-Infinite Robust Trajectory Planning problem:

$$\underset{\bar{x}^n_{T+1}, \bar{u}_T}{\text{minimize}} \quad J(\bar{u}_T)$$

s.t. <u>Dynamics with perturbations:</u>

$$x_{t+1} = \Phi_{t+1,0} x_0^n + \bar{B}_t \bar{u}_t + \Phi_{t+1,0} \Delta x_0 + \bar{E}_t \bar{n}_t \qquad t = 0, \ldots, N$$

<u>SIP state constraints:</u>

$$H_{t+1} x_{t+1} \leq h_{t+1} \qquad \forall \bar{n}_t \ \text{ s.t. } 0 \leq \bar{n}_t \leq \bar{f}(\bar{x}^n_t, \bar{u}_t) \quad t = 0, \ldots, N$$

Analytical Results

SIP State Constraint - Reformulation

- **SIP state constraints** satisfied **robustly** if

$$\max_{0 \leq \bar{n}_t \leq \bar{f}(\bar{x}_t^n, \bar{u}_t)} H_{t+1,i}\, \bar{E}_t \bar{n}_t \leq M_{RHS,i}, \qquad i = 1, \ldots, n_h$$

where

$$H_{t+1} = \begin{bmatrix} H_{t+1,1} \\ \vdots \\ H_{t+1,n_h} \end{bmatrix}$$

$$M_{RHS,i} := h_{t+1,i} - H_{t+1,i}(\Phi_{t+1,0} x_0^n + \bar{B}_t \bar{u}_t + \Phi_{t+1,0} \Delta x_0)$$

Analytical Results

SIP State Constraint - Reformulation

- **Dual** of the **maximization problem**:

$$\begin{array}{ll} \min_{\lambda_{t,i}} & \lambda_{t,i}^T \bar{f}(\bar{x}_t^n, \bar{u}_t) \\ \text{s.t.} & \lambda_{t,i} \geq 0, \quad \lambda_{t,i}^T \geq H_{t+1,i}\, \bar{E}_t \end{array}$$

- **Feasible dual cost** bounds **the primal**.
  $\Rightarrow$ **SIP state constraints** satisfied if

$$\lambda_{t,i}^T \bar{f}(\bar{x}_t^n, \bar{u}_t) \leq M_{RHS,i}$$

$$\lambda_{t,i} \geq 0, \quad \lambda_{t,i}^T \geq H_{t+1,i}\, \bar{E}_t$$

for all $i \in 1, \ldots, n_h$.

Analytical Results

SIP State Constraint - Reformulation

- **Dual variables** in stacked form:

$$\Lambda_t = \begin{bmatrix} \lambda_{t,1}^T \\ \vdots \\ \lambda_{t,n_h}^T \end{bmatrix}$$

- **State constraints** in stacked form:

$$\Lambda_t \geq H_{t+1}\, \bar{E}_t, \qquad \Lambda_t \geq 0,$$
$$\Lambda_t \bar{f}(\bar{x}_t^n, \bar{u}_t) \leq M_{RHS}.$$

- **Biconvex** in Λ_t, $\bar{x}_t^n$, $\bar{u}_t$.
- Minimum components of $\Lambda_t \Rightarrow$ minimum **dual cost** $\Rightarrow$ no **conservatism**

Analytical Results

Closed-form solution

- **Strong duality** holds for the **constraint reformulation**

  $\Rightarrow$ State Constraints satisfied

  - **robustly**
  - **with no conservatism**

  for the following choice of dual variable Λ:

$$\Lambda_t^* = \max\{0, H_{t+1}\,\bar{E}_t\}$$

Analytical Results

Convex trajectory planning problem

Convex Reduction $\Rightarrow$ **Convexified** Robust Trajectory Planning problem

$$\begin{aligned} \underset{\bar{x}^n_{T+1}, \bar{u}_T}{\text{minimize}} \quad & J(\bar{u}_T) \\ \text{s.t.} \quad & \underline{\text{Dynamics:}} \\ & x^n_{t+1} = \Phi_{t+1,0} x^n_0 + \bar{B}_t \bar{u}_t \quad t = 0, \ldots, N \\ & \underline{\text{State constraints:}} \\ & \Lambda^*_t \bar{f}(\bar{x}^n_t, \bar{u}_t) \leq M_{RHS} \qquad t = 0, \ldots, N \end{aligned}$$

Example Simulation

Spacecraft Proximity Operations

- **Clohessy-Wiltshire** equations for dynamics;
- S/C must **navigate** from (relative) stationary position into a **terminal bounding box**, s.t. keep-out plane constraint [12], [13]

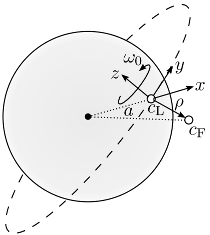


Figure: Schematics of the problem[1]

[1] Malyuta, D., Açıkmeşe, B. and Cacan, M. "Robust Model Predictive Control for Linear Systems with State and Input Dependent Uncertainties", ACC 2019

Example Simulation

Spacecraft Proximity Operations

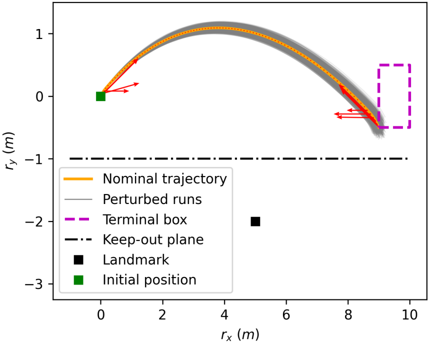


Figure: No planning for perturbation; many trajectories unsafe.

Perturbations:

- **Control uncertainty**: error proportional to **control magnitude** (Gates' model)
- **Navigation uncertainty**: error proportional to **distance from landmark:**

$$0 \leq n \leq 2(r - r_{lm})^2$$

Example Simulation

Spacecraft Proximity Operations

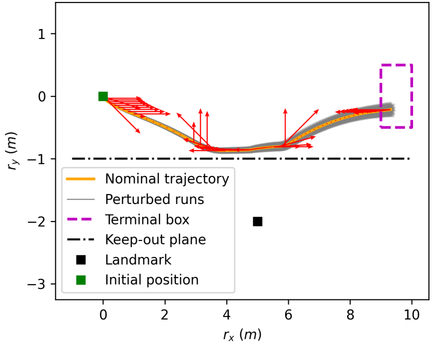


Figure: Planning for perturbation; all trajectories safe.

Perturbations:

- **Control uncertainty**: error proportional to **control magnitude** (Gates' model)
- **Navigation uncertainty**: error proportional to **distance from landmark**

$$0 \leq n \leq 2(r - r_{lm})^2$$

Contents

Problem Formulation

Linear System with Feedback

- Dynamics with perturbations as before:

$$x_{t+1} = A_t x_t + B_t u_t + E_t n_t \qquad\qquad t = 0, \dots, N$$

but

- **Control** u_t is decomposed into **open-loop** v_t and a **linear state feedback** $K_t \Delta x_t$:

$$u_t = v_t + K_t \Delta x_t$$

- Nominal and deviation dynamics **with feedback** in resursive form:

$$x^n_{t+1} = A_t x^n_t + B_t v_t,$$
$$\Delta x_{t+1} = (A_t + B_t K_t)\Delta x_t + E_t n_t.$$

Problem Formulation

Linear System with Feedback

- Control in stacked form:

$$\bar{u}_t = \bar{v}_t + \bar{K}_t \Delta\bar{x}_t$$

given

$$\bar{K}_t \coloneqq \mathrm{diag}(K_0, \ldots, K_t),\ \Delta\bar{x}_t \coloneqq \begin{bmatrix} \Delta x_0 \\ \vdots \\ \Delta x_t \end{bmatrix},\ \bar{v}_t \coloneqq \begin{bmatrix} v_0 \\ \vdots \\ v_t \end{bmatrix}$$

Problem Formulation

Linear System with Feedback

- Matrices:

$$\bar{\Phi}_{t,0} := \begin{bmatrix} I \\ \vdots \\ \Phi_{t,0} \end{bmatrix} \quad \bar{\bar{E}}_t := \begin{bmatrix} \bar{E}_0 & 0 \\ \vdots & \vdots \\ \bar{E}_t & 0 \end{bmatrix}$$

- Nominal and deviation dynamics **with feedback** in stacked form:

$$x_{t+1}^n = \Phi_{t+1,0} x_0^n + \bar{B}_t \bar{v}_t,$$
$$\Delta x_{t+1} = (\Phi_{t+1,0} + \bar{B}_t \bar{K}_t \bar{\Phi}_{t,0}) \Delta x_0 + (\bar{E}_t + \bar{B}_t \bar{K}_t \bar{\bar{E}}_{t-1}) \bar{n}_t.$$

Problem Formulation

Perturbation, Cost and Constraints

- As before:
  - **Nominal state and control**-dependent perturbation;

$$0 \leq n_t \leq f(x_t^n, v_t)$$

  - **SIP state constraints**:

$$H_{t+1}x_{t+1} \leq h_{t+1} \qquad \forall \bar{n}_t$$

- **Convex** cost function, treated with SIP constraint:

$$J(\bar{u}_T) \leq \mathcal{J} \qquad \forall \bar{n}_T$$

Problem Formulation

Trajectory planning problem

Semi-Infinite Robust Trajectory Planning problem:

$$\underset{\bar{x}^n_{T+1},\bar{v}_T,\bar{K}_T,\mathcal{J}}{\text{minimize}} \quad \mathcal{J}$$

s.t.

Nominal dynamics:

$$x^n_{t+1} = \Phi_{t+1,0} x^n_0 + \bar{B}_t \bar{v}_t \qquad t = 0, \dots, N$$

Deviation dynamics:

$$\Delta x_{t+1} = (\Phi_{t+1,0} + \bar{B}_t \bar{K}_t \bar{\Phi}_{t,0}) \Delta x_0 + (\bar{E}_t + \bar{B}_t \bar{K}_t \bar{\bar{E}}_{t-1}) \bar{n}_t \qquad t = 0, .$$

SIP state constraints:

$$H_{t+1} x_{t+1} \leq h_{t+1} \qquad \forall \bar{n}_t \ \text{ s.t. } 0 \leq \bar{n}_t \leq \bar{f}(\bar{x}^n_t, \bar{v}_t) \qquad t = 0, \dots, N$$

SIP cost constraint:

$$J(\bar{u}_T) \leq \mathcal{J} \qquad \forall \bar{n}_T \ \text{ s.t. } 0 \leq \bar{n}_T \leq \bar{f}(\bar{x}^n_T, \bar{v}_T)$$

Analytical Results
SIP State Constraint - Reformulation

- **SIP state constraints** satisfied if

$$\begin{aligned} \max_{\bar{n}_t} \quad & H_{t+1,i}\,(\bar{E}_t + \bar{B}_t \bar{K}_t \bar{\bar{E}}_{t-1})\bar{n}_t \leq M_{RHS,i} \quad i = 1, \ldots, n_h \\ \text{s.t.} \quad & 0 \leq \bar{n}_t \leq \bar{f}(\bar{x}_t^n, \bar{v}_t) \end{aligned}$$

where

$$M_{RHS,i} := h_{t+1,i} - H_{t+1,i}(\Phi_{t+1,0} x_0^n + \bar{B}_t \bar{v}_t + (\Phi_{t+1,0} + \bar{B}_t \bar{K}_t \bar{\Phi}_{t,0})\Delta x_0)$$

Analytical Results

SIP State Constraint - Reformulation

- **Dualization**
  $\Rightarrow$ Equivalent **state constraints** in stacked form:

$$\Lambda_t \geq H_{t+1}\left(\bar{E}_t + \bar{B}_t \bar{K}_t \bar{\bar{E}}_{t-1}\right), \quad (1)$$

$$\Lambda_t \geq 0, \quad (2)$$

$$\Lambda_t \bar{f}(\bar{x}_t^n, \bar{v}_t) \leq M_{RHS}. \quad (3)$$

- **Biconvex** in Λ_t, $\bar{x}_t^n$, $\bar{v}_t$, minimum components of Λ_t
  $\Rightarrow$ no **conservatism**.
- What about the gain $\bar{K}_t$?

Analytical Results

Feedback Gain Synthesis Step

- System is **stable** iff exist G and **symmetric** P such that [2]

$$\begin{bmatrix} P & (A_t + B_t K_t)^T G_t^T \\ G_t(A_t + B_t K_t) & G_t + G_t^T - P \end{bmatrix} \succ 0 \tag{4}$$

Algorithm Feedback Gain Synthesis

Input: $K^{(0)}$, ε_G

```
k ← 0
while ||P^(k) − G^(k)|| > ε_G do
    Fix K = K^(k), minimize ||P − G||_2 subject to (4)
    Fix G = G^(k), minimize 1^⊤ Λ 1 subject to (1), (2), (4)
    k ← k + 1
end while
Return K^(k)
```

Analytical Results

SIP Cost Constraint - Reformulation

- **Cost** $\partial_{\bar{n}_T} J \, \bar{n}_T$ **bounded** by $\mathcal{J}_n$ if :

$$\begin{aligned} \max_{\bar{n}_T} \quad & \partial_{\bar{n}_T} J \, \bar{n}_T \le \mathcal{J}_n \\ \text{s.t.} \quad & 0 \le \bar{n}_T \le \bar{f}(\bar{x}_T^n, \bar{v}_T) \end{aligned}$$

- **Dualizing** and **expressing constraints** in stacked form:

$$\begin{aligned} &\Lambda_T^J \ge \partial_{\bar{n}_T} J, \\ &\Lambda_T^J \ge 0, \\ &\Lambda_T^J \bar{f}(\bar{x}_T^n, \bar{v}_T) \le \mathcal{J}_n \end{aligned}$$

- Minimum components of $\Lambda_T^J \Rightarrow$ no **conservatism**.

Analytical Results

Closed-form solution

- SIP State Constraints satisfied for the choice of Λ_t

$$\Lambda_t^* = \max\{0, H_{t+1}\left(\bar{E}_t + \bar{B}_t \bar{K}_t \bar{\bar{E}}_{t-1}\right)\}$$

- **Uncertain cost** bounded
  - **with no conservatism**

  for the choice of Λ_T^J:

$$\Lambda_T^{J*} = \max\{0, \partial_{\bar{n}_T} J\}$$

Analytical Results

Trajectory Planning Step

Convexified Robust Trajectory Planning problem

$$
\begin{aligned}
\underset{\bar{x}^n_{T+1},\bar{v}_T,\mathcal{J},\mathcal{J}_n}{\text{minimize}} \quad & \mathcal{J} \\
\text{s.t.} \quad & \underline{\text{Dynamics:}} \\
& x^n_{t+1} = \Phi_{t+1,0} x^n_0 + \bar{B}_t \bar{v}_t \qquad t = 0, \dots, N \\
& \underline{\text{State constraints:}} \\
& \Lambda^*_t \bar{f}(\bar{x}^n_t, \bar{v}_t) \leq M_{RHS} \qquad t = 0, \dots, N \\
& \underline{\text{Cost function:}} \\
& \Lambda^{J*}_T \bar{f}(\bar{x}^n_T, \bar{v}_T) \leq \mathcal{J}_n \\
& J\left(\bar{v}_T + \partial_{\Delta x_0} \bar{u}_T \, \Delta x_0\right) + \mathcal{J}_n \leq \mathcal{J}
\end{aligned}
$$

Analytical Results

Complete algorithm

Algorithm Convex Robust Trajectory Planning and control

1: **Compute** Stabilizing control guess $K^{(0)}$
2: **Solve** Feedback Gain Synthesis
3: **Compute** $\Lambda_t^*, \Lambda_T^{J*}$
4: **Solve** Convexified robust trajectory planning
5: **Return** $\bar{x}_{T+1}^n, \bar{v}_T, \bar{K}_T, \mathcal{J}$

- **Control synthesis** and **trajectory optimization** performed **sequentially**

Example Simulation

Double Integrator with K synthesis

- 2-DoF double integrator; difference between **open-loop** vs. **closed-loop** Monte Carlo simulations.
- Vehicle must **navigate** from stationary position into a **terminal bounding box**, subject to
  - perturbation **increasing with distance** from x-axis
  - optimized **linear feedback** K

Example Simulation

Double Integrator with K synthesis

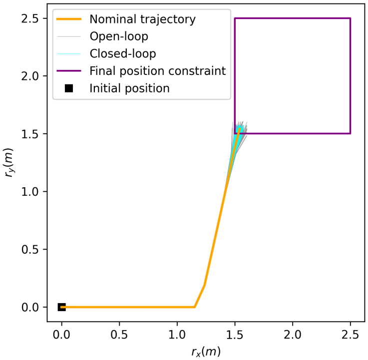


Figure: Dispersed trajectories with feedback

Perturbations [14]:

- **Navigation uncertainty**: error proportional to **distance from y-plane:**

$$0 \leq n \leq 2r_y^2$$

Example Simulation

Double Integrator with K synthesis

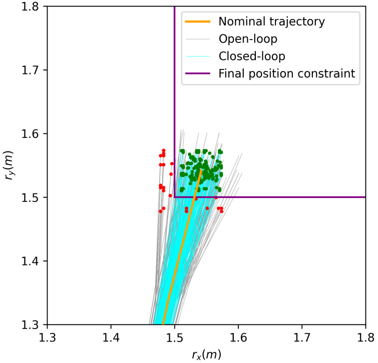


Figure: Control ensures constraint satisfaction

Perturbations:

- **Navigation uncertainty**: error proportional to **distance from y-plane:**

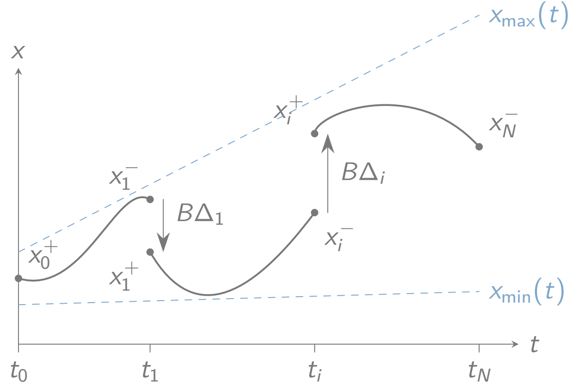
$$0 \leq n \leq 2r_y^2$$

Contents

Problem Formulation

Overview

Apply an impulse at each time instant t_i.
Satisfy **state constraints for all times $t \in [t_0, t_N]$**.

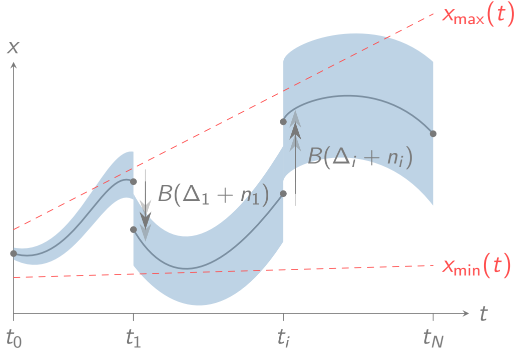


Problem Formulation

Overview

Uncertainty **grows**:

- **during** each interval
- **at** each impulse



Problem Formulation

Dynamics

- **Continuous-time (CT) nonlinear** dynamical system with endogenous **uncertain impulsive controls** [16].
- **CT** dynamics with **uncertain** initial conditions:

$$\dot{x}(t) = f(x(t)), \qquad x(t_0^+) = \bar{x}_0^- + B_0 n_{x,0}$$

  where $\bar{x}_0^-$ is assigned.
- **Uncertain impulsive** controls:

$$x(t_i^+) = x(t_i^-) + B(\Delta_i + n_i) \qquad i = 0, \ldots, N$$

Problem Formulation

Nominal and deviation dynamics

- We decompose **state** into **nominal** $\hat{x}$ and **deviation states** δx:

$$x(t) = \hat{x}(t) + \delta x(t)$$

- Nominal state and deviation **dynamics**:

$$\begin{aligned} \dot{\hat{x}}(t) &= f(\hat{x}(t)) \\ \delta\dot{x}(t) &= f(x(t)) - f(\hat{x}(t)) \\ \hat{x}(t_i^+) &= \hat{x}(t_i^-) + B\Delta_i \\ \delta x(t_i^+) &= \delta x(t_i^-) + Bn_i, \quad \delta x(t_0^-) = B_0 n_{x,0} \end{aligned}$$

Problem Formulation

Perturbation, Constraints, Cost

- **Nominal state and control**-dependent perturbations:

$$\begin{aligned} 0 &\leq n_i \leq \beta_{\Delta,i}(\hat{x}(t_i^-), \Delta_i) \\ 0 &\leq n_{x,0} \leq \beta_x(\hat{x}(t_0^-)) \end{aligned}$$

  where β_Δ, β_x are **elementwise nonnegative** functions.

- **SIP continuous-time linear** state constraints

$$H\,x(t) \leq h \qquad \forall \boldsymbol{n} := \left[\; n_0^\top \; n_1^\top \; \dots \; n_N^\top \;\right]^\top$$

- **Nominal** cost

$$L(\hat{x}_N)$$

Problem Formulation

Nonlinear Robust trajectory planning problem

Semi-Infinite Nonlinear Robust Trajectory Planning problem:

$$
\begin{aligned}
\underset{\hat{x},\Delta}{\text{minimize}} \quad & L(\hat{x}_N) \\
\text{s.t.} \quad & \underline{\text{Dynamics:}} \\
& \dot{x}(t) = f(x(t)) \\
& \underline{\text{Uncertain impulsive controls:}} \\
& x(t_i^+) = x(t_i^-) + B(\Delta_i + n_i) \quad i = 0, \dots, N \\
& \underline{\text{Uncertain initial conditions:}} \\
& x(t_0^-) = \bar{x}_0^- + B_0 n_{x,0} \\
& \underline{\text{SIP state constraints:}} \\
& Hx(t) \leq h \qquad \forall \boldsymbol{n} \ \text{ s.t. } \ 0 \leq n_i \leq \beta_{\Delta,i}(\hat{x}(t_i^-), \Delta_i) \\
& \qquad\qquad\qquad\qquad\quad\;\; 0 \leq n_{x,0} \leq \beta_x(\hat{x}(t_0^-))
\end{aligned}
$$

Analytical Results

Uncertain State Constraint - Reformulation

- **SIP approximate state constraints** satisfied **robustly** at each time t if

$$\begin{aligned} \max_{\boldsymbol{n}} \quad & H_i \boldsymbol{E}(t)\boldsymbol{n} \leq M_{RHS,i} \\ \text{s.t.} \quad & 0 \leq \boldsymbol{n} \leq \beta(\hat{\boldsymbol{x}}^-, \boldsymbol{\Delta}) \end{aligned}$$

where

$$\beta := \begin{bmatrix} \beta_x \\ \beta_{\Delta,0} \\ \vdots \\ \beta_{\Delta,N} \end{bmatrix}, \; H = \begin{bmatrix} H_1 \\ \vdots \\ H_{n_h} \end{bmatrix}, \; M_{RHS} = h - H\hat{x}(t),$$

$$\boldsymbol{E}(t) := \left[\frac{\partial x(t)}{\partial n_{x,0}} \; \frac{\partial x(t)}{\partial n_1} \cdots \frac{\partial x(t)}{\partial n_N} \right]$$

Analytical Results

Uncertain State Constraint - Reformulation

- Using **duality**, SIP constraint is satisfied **for all times t** if

$$\Lambda(t) \geq H\boldsymbol{E}(t), \quad \Lambda(t) \geq 0,$$
$$\Lambda(t)\beta(\hat{\boldsymbol{x}}^{-}, \boldsymbol{\Delta}) \leq M_{RHS}$$

- **Constraint duals** are minimized

$$\Lambda^{*}(t) = \max\{0, H\boldsymbol{E}(t)\}$$

Analytical Results - Nonlinear Case

Uncertain State Constraint - CT reformulation

- **Augment** state[1] with $\xi(t)$, with dynamics

$$f_\xi(\hat{\mathbf{x}}, \Delta) := \max\{0, \Lambda^*(t)\beta(\hat{\mathbf{x}}^-, \Delta) - M_{RHS}\}^2$$

- Constraints satisfied iff **isoperimetric boundary conditions** satisfied:

$$\begin{aligned} \dot{\xi}(t) &= f_\xi(\hat{\mathbf{x}}, \Delta) \\ \xi_i^+ - \xi_{i+1}^- &= 0 \qquad\qquad i = 0, \ldots, N-1 \end{aligned}$$

[1] Elango, P., Luo, D., Kamath, A. G., Uzun, S. Kim, T. and Açıkmeşe, B. "Continuous-Time Successive Convexification for Constrained Trajectory Optimization." Automatica, 2025.

Analytical Results

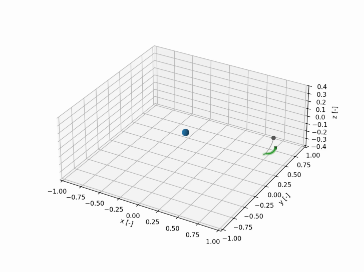
Nonlinear CT-Robust trajectory planning problem

Nonlinear **CT-Robust** Trajectory Planning problem:

$$
\begin{aligned}
\underset{\hat{x},\Delta}{\text{minimize}} \quad & L(\hat{x}_N) \\
\text{s.t.} \quad & \underline{\text{Dynamics:}} \\
& \dot{\hat{x}}(t) = f(\hat{x}(t)) \\
& \underline{\text{Impulsive controls:}} \\
& \hat{x}_i^+ = \hat{x}_i^- + B\Delta_i \quad i = 0, \ldots, N \\
& \underline{\text{Initial conditions:}} \\
& \hat{x}(t_0^-) = \bar{x}_0^- \\
& \underline{\text{State constraints:}} \\
& \dot{\xi}(t) = f_\xi(\hat{\boldsymbol{x}}, \boldsymbol{\Delta}) \\
& \xi_i^+ - \xi_{i+1}^- = 0 \qquad\qquad i = 0, \ldots, N-1
\end{aligned}
$$

Example Simulation

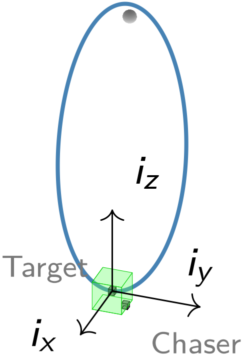
Nonlinear Near-Rectilinear Halo Orbit Dynamics



Example Simulation

Nonlinear Near-Rectilinear Halo Orbit Dynamics

- **3-DoF relative dynamics** on Near-Rectilinear Halo Orbit
- **Chaser** must loiter **for as long as possible** in a box **around target**.
- **Uncertain impulse** proportional to
  - **control magnitude** (Gates' model)
  - **position on orbit** (high at **perilune**, low at **apolune**)



Example Simulation

Nonlinear Near-Rectilinear Halo Orbit Dynamics

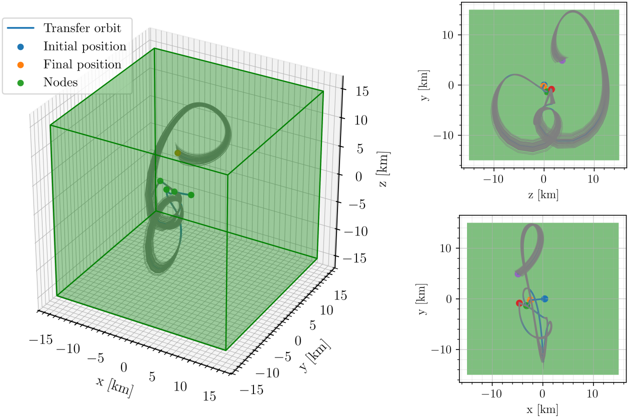


Figure: Dispersed trajectories around the target - Dots are impulse locations

Example Simulation

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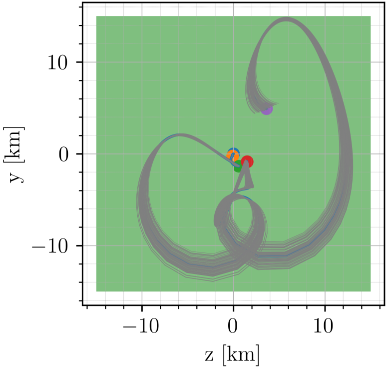

- Initial offset along **Earth-Moon** axis
- Impulses applied where **position uncertain** is **low**
- All trajectories **inside** the box **at all times**

Contents

Conclusions

Q0: How to incorporate **robust constraint satisfaction**?

- **Duality** can bypass **Semi-Infinite** formulations of robust trajectory optimization problems.

Q1: How to incorporate **robust constraint satisfaction** without **conservatism**?

- **Duality** enables **robust constraint satisfaction** for both linear and nonlinear dynamical systems;
- **Endogenous** perturbations can be treated with **no conservatism** under mild modeling assumptions;

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